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AN EFFECT OF MAGNETIC PARAMETER ON MHD BOUNDARY LAYER MODEL FOR NEWTONIAN POWER-LAW FLUIDS USING COLLOCATION METHOD

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ABSTRACT

In the present paper, the equation of Magneto hydrodynamic boundary layer model for Newtonian power law fluid has been analyzed in the presence of transverse magnetic field for two cases, (i) when plate is stationary (ii) when the fluid and plate moves in same direction and with same velocity. Here we solve the governing nonlinear differential equation with their associated boundary conditions using Spline functions due to Blue. The beauty of this method is we can solve nonlinear problem directly, without converting to the linear form. Method description is explained and graphical results describing the displacement and velocity of the flow are presented in this work.

***KEYWORDS:** Power-Law Fluids, Magnetic Field, Nonlinear Differential Equation, Collocation Method & Spline Functions*

Nomenclature:

***MHD** - Magneto hydrodynamics*

u, v ***- Velocity components in*** x, y ***directions respectively***

τ_{xy} ***- Shear stress***

ρ ***- Field density***

ψ ***- Stream function***

σ ***- Electrical conductivity***

K ***-consistency coefficient***

v ***- Shear rate***

B_0 ***- Uniform strength of magnetic field***

n ***- Power-law index***

η ***- Similarity variable***

$\in$ ***- Velocity parameter***

M ***- Magnetic parameter***

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Original Article

1. INTRODUCTION

Researchers in mechanical discipline have curiosity and interest to find the solution of nonlinear differential equations for governing mathematical model. Applications of boundary layer flow are in textile and paper industry, manufacture of sheets, crystalline materials, spinning of fibers. [1]Wu (1973) has studied the effects of suction or injection on MHD boundary layer flow. [2] Takhar et.al. (1987) studied a MHD asymmetric flow over a semi-infinite moving surface [3] Mahapatra and Gupta (2001) studied a steady two- dimensional stagnation-point flow by a uniform transverse magnetic field. [4] Jean-David Hoernel (2008) has been investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow. [5] Rajput (2014) study of MHD boundary layer flow of non-newtonian power law fluid. Pakdemirli [6] derived the boundary layer equations of power-fluids. Cubic Spline functions were used to solve second order Föppl-Hencky equation by [7] Pandya et al. (2010).

The object of the present paper is to study the effect of magnetic parameter on MHD flow for two cases, when plate is at rest and fluid and plate moves in same direction with same velocity. The governing equation is nonlinear differential equations, which is solved by using spline collocation method due to blue [8]. In this way, the paper has been organized as follows. In section 2, problem formulation is given, in section 3, method description and approximate solution by using collocation method and results and discussion are presented in section 4. Section 5 includes the conclusion.

2. GOVERNING EQUATIONS

Here we studied the two-dimensional laminar boundary layer flow of a viscous, incompressible and electro conducting power law fluid past a continuously moving surface passing through with constant U_w in the same direction to the free stream velocity U_∞. The x - axis extends parallel to the plate and y - axis perpendicular to the x - axis. A magnetic field of uniform strength B_0 is applied in the positive y - direction, which produce the magnetic field in the x - direction. The boundary layer equations governing the flow in a power-law fluid are

$$\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}-\frac{\sigma B_0^2}{\rho}u \quad \text{s} \tag{2.2}$$

Where u , v are the velocity components along x and y coordinates, τ_{xy} is the shear stress and ρ is the fluid density.

with the boundary conditions:

$$\begin{aligned} &y=0: \quad u=U_w, \quad v=0 \\ &y=\infty: \quad u=U_\infty \end{aligned} \tag{2.3}$$

We apply power-law relation between the shear stress and the shear rate by

$$v = -\frac{\partial \psi}{\partial x} \; \tau_{xy} = K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y}$$

Where $\gamma \left| \frac{\partial u}{\partial y} \right|^{n-1}$ denotes the kinematic viscosity, K is the consistency coefficient $\gamma = \frac{K}{\rho}$ and n is the power-law index, for $n<1$ pseudo plastic, for $n=1$ the fluid is Newtonian, $n>1$ for dilatants fluid. The equation (8) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(\gamma \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0^2}{\rho} u \tag{2.4}$$

Introducing the stream function $\psi(x, y)$ such that $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$, which satisfy the continuity equation (2.1). Govind R. Rajput et al [5] converted partial differential equation into nonlinear ordinary differential equations using Group theoretic method. They considered the following transformation:

$$\psi(x, y) = ax^{\alpha} f(\eta), \quad \eta = b\frac{y}{x^b} \tag{2.5}$$

Where a, b, α and β are real numbers, η is similarity variable, $f(n)$ is the transformed dimensionless stream function.

Applying this similarity variable η they derive

$$\begin{aligned}
\psi_x &= ax^{\alpha-1}[\alpha f - \eta\beta f^{'}] \\
\psi_y &= abf^{'} x^{\alpha-\beta} \\
\psi_{yy} &= ab^2 x^{\alpha-2\beta} f^{"} \\
\psi_{yx} &= abx^{\alpha-\beta-1}[\alpha f^{'} - \beta f^{'} - \beta\eta f^{"}]
\end{aligned} \tag{2.6}$$

Using the equation (2.5) along with (2.6) into (2.4) they get transformed into nonlinear ordinary differential of the form

$$\left(\left| f^{"} \right|^{n-1} f^{"} \right)^{'} - Mf^{'} + \frac{1}{n+1} ff^{"} = 0 \left(\left| f^{"} \right|^{n-1} f^{"} \right)^{'} - Mf^{'} + \frac{1}{n+1} ff^{"} = 0 \tag{2.7}$$

With the transformed boundary conditions:

$$f(0) = 0, \quad f^{'}(0) = \in, \; f^{'}(\infty) = 1 \tag{2.8}$$

n =1 gives Newtonian fluids, then the equation (2.7) becomes

$$f''' - Mf' + \frac{1}{2} ff'' = 0 \tag{2.9}$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = \in, \ f'(\infty) = 1 \tag{2.10}$$

Where $\in = \dfrac{U_w}{U_\infty}$ is the velocity parameter and $M = \dfrac{\sigma B_0^2}{\rho U_\infty} x$ is the magnetic parameter. Here note that when $\in = 0$ plate is stationary and $\in = 1$ plate and fluid moves same direction and same velocity.

3. QUARTIC SPLINE BLUE METHOD

For three points boundary value problems are

Let $s_i(x)$ be quartic spline function in $[x_{i-1}, x_i]$. Conditions for natural splines are

- $s_i(x)$ Almost quartic in each subinterval $[x_{i-1}, x_i]$.
- $s_i(x_i) = y_i$, for i = 0,1,2,........,n.
- $s_i(x_i)$, $s_i'(x_i)$, $s_i''(x_i)$, $s'''(x_i)$ are continuous in $[x_0, x_n]$.
- $s_i'''(x_0) = s_i'''(x_n) = 0$.

Here spline third derivative must be linear in $[x_{i-1}, x_i]$. So,

$$s_i'''(x_i) = \frac{1}{h_i}\left[(x_i - x) y_{i-1}''' + (x - x_{i-1}) y_i'''\right] \tag{3.1}$$

Where $h_i = x_i - x_{i-1}$ and $s'''(x_i) = y_i'''$

Integrate (3.1) twice with respect to x

$$s_i'(x) = \frac{1}{h_i}\left[\frac{(x_i - x)^3}{6} y_{i-1}''' + \frac{(x - x_{i-1})^3}{6} y_i'''\right] + c_i(x_i - x) + d_i(x - x_{i-1}).$$

Where use $s_i'(x_{i-1}) = y_{i-1}'$ and $s_i'(x_i) = y_i'$ in (3.1), we get constants c_i and d_i

$$c_i = \frac{1}{h_i}\left(y_{i-1}' - \frac{h_i^2}{6} y_{i-1}'''\right) \quad \text{and} \quad d_i = \frac{1}{h_i}\left(y_i' - \frac{h_i^2}{6} y_i'''\right)$$

So

$$s_i^{'}(x) = \frac{1}{h_i}\left[\frac{(x_i-x)^3}{6}y_{i-1}^{'''}+\frac{(x-x_{i-1})^3}{6}y_i^{'''}\right]+\frac{1}{h_i}\left(y_{i-1}^{'}-\frac{h_i^2}{6}y_{i-1}^{'''}\right)(x_i-x)+$$
$$\frac{1}{h_i}\left(y_i^{'}-\frac{h_i^2}{6}yi^{'''}\right)(x-x_{i-1}). \tag{3.2}$$

Integrate (3.2), once with respect to x,

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i-x)^4}{24}y_{i-1}^{'''}+\frac{(x-x_{i-1})^4}{24}y_i^{'''}\right]-\frac{1}{h_i}\left(y_{i-1}^{'}-\frac{h_i^2}{6}y_{i-1}^{'''}\right)\frac{(x_i-x)^2}{2}+$$
$$+\frac{1}{h_i}\left(y_i^{'}-\frac{h_i^2}{6}yi^{'''}\right)\frac{(x-x_{i-1})^2}{2}+e_i. \tag{3.3}$$

Take $s_i(x_{i-1})=y_{i-1}$, we get constants e_i

where $e_i = y_{i-1}-\frac{h^3}{8}y_{i-1}^{'''}+\frac{h}{2}y_{i-1}^{'}.$

Substitute e_i in (3.3), we get

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i-x)^4}{24}y_{i-1}^{'''}+\frac{(x-x_{i-1})^4}{24}y_i^{'''}\right]-\frac{1}{h_i}\left(y_{i-1}^{'}-\frac{h_i^2}{6}y_{i-1}^{'''}\right)\frac{(x_i-x)^2}{2}+$$
$$+\frac{1}{h_i}\left(y_i^{'}-\frac{h_i^2}{6}yi^{'''}\right)\frac{(x-x_{i-1})^2}{2}+y_{i-1}-\frac{h^3}{8}y_{i-1}^{'''}+\frac{h}{2}y_{i-1}^{'}. \tag{3.4}$$

Here $s_i^{''}(x_i^-)=s_{i+1}^{''}(x_i^+)$ and for equal intervals we have,

$$y_{i+1}^{'}-2y_i^{'}+y_{i-1}^{'}=\frac{h^2}{6}(y_{i+1}^{'''}+4y_i^{'''}+y_{i-1}^{'''}) \tag{3.5}$$

And for $s_i(x_i^-)=s_{i+1}(x_i^+)$ and for equal intervals we have,

$$y_{i-1}-y_i=-\frac{h}{2}\left(y_i^{'}+y_{i-1}^{'}\right)+\frac{h^3}{24}(3y_{i-1}^{'''}-y_i^{'''}) \tag{3.6}$$

3.1 Solution of the Problem by using Spline Function Due to Blue

We solve the nonlinear ordinary differential equation

$$f''' - Mf' + \frac{1}{2} ff'' = 0 \tag{3.1.1}$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = \in, \ f'(1) = 1 \tag{3.1.2}$$

Solve this for two cases when $\in = 0$ plate is stationary and $\in = 1$ plate and fluid moves same direction and same velocity.

Case (i) $\in = 0$ **plate is stationary**

To obtain the spline solution, we begin with an assumed function $f(\eta) = \frac{1}{2}\eta^2$, which satisfy given boundary conditions (3.1.2). To find the solution of equation (3.1.1) along with boundary conditions (3.1.2), we use $f(\eta) = \frac{1}{2}\eta^2$, equations (3.1.1) and (3.5). Put h=0.1, we get different values of y_i' for $i = 1, 2, 3, 4$.

To find the final solution we use (3.6) for different values of $i = 1, 2, 3, 4$ respectively

Thus, we obtain the system of equations as follows

$$\begin{aligned}
y_0 - y_1 &= -\frac{h}{2}[y_1' + y_0'] + \frac{h^3}{24}[-y_1''' + 3y_0'''] \\
y_1 - y_2 &= -\frac{h}{2}[y_2' + y_1'] + \frac{h^3}{24}[-y_2''' + 3y_1'''] \\
y_2 - y_3 &= -\frac{h}{2}[y_3' + y_2'] + \frac{h^3}{24}[-y_3''' + 3y_2'''] \\
y_3 - y_4 &= -\frac{h}{2}[y_4' + y_3'] + \frac{h^3}{24}[-y_4''' + 3y_3''']
\end{aligned} \tag{3.1.3}$$

Substitute y_i' and y_i''' for $h = 0.1$ in (3.1.3). We get four unknown and four equations. Solving that system using Matlab, we get solution graph as follows:

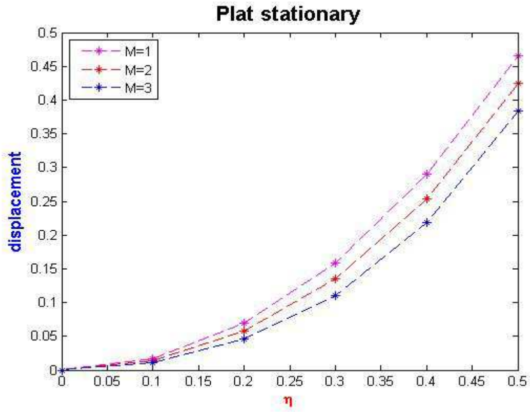

Figure 1: $f(\eta)$ versus η for different Values of M

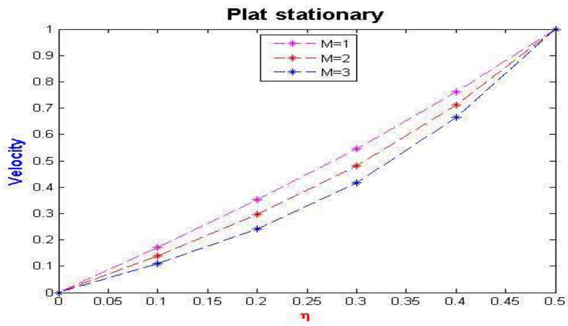

Figure 2: $f^{'}(\eta)$ versus η for different Values of M

Case (ii) $\in =1$ plate and fluid moves same direction and same velocity

To obtain the spline solution, we begin with an assumed function $f(\eta)=\eta$ which satisfy given boundary conditions (3.1.2). To find the solution of equation (3.1.1) along with boundary conditions (3.1.2). First we use $f(\eta)=\eta$, equation (3.1.1) and (3.5) and $h=0.1$ to get different values of $y_i^{'}$ for $i=1,2,3,4$.

To find the final solution we use (3.6) for different values of $i=1,2,3,4$ respectively and get the system of equations

$$y_0-y_1=-\frac{h}{2}[y_1^{'}+y_0^{'}]+\frac{h^3}{24}[-y_1^{'''}+3y_0^{'''}]$$
$$y_1-y_2=-\frac{h}{2}[y_2^{'}+y_1^{'}]+\frac{h^3}{24}[-y_2^{'''}+3y_1^{'''}]$$
$$y_2-y_3=-\frac{h}{2}[y_3^{'}+y_2^{'}]+\frac{h^3}{24}[-y_3^{'''}+3y_2^{'''}]$$
$$y_3-y_4=-\frac{h}{2}[y_4^{'}+y_3^{'}]+\frac{h^3}{24}[-y_4^{'''}+3y_3^{'''}] \quad (3.1.4)$$

Substitute $y_i^{'}$ and $y_i^{'''}$ for $h=0.1$ in (3.1.4). We get four unknown and four equations, solving which using

Matlab we get solution graph as follows:

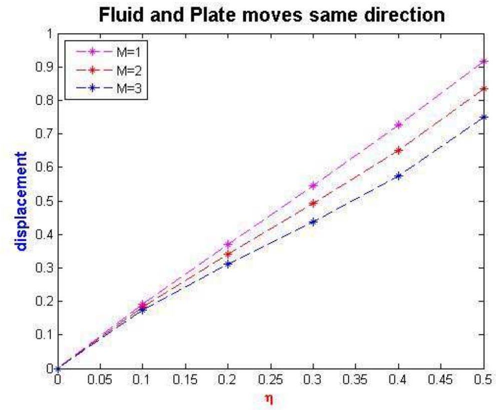

Figure 3: $f(\eta)$ versus η for different values of M

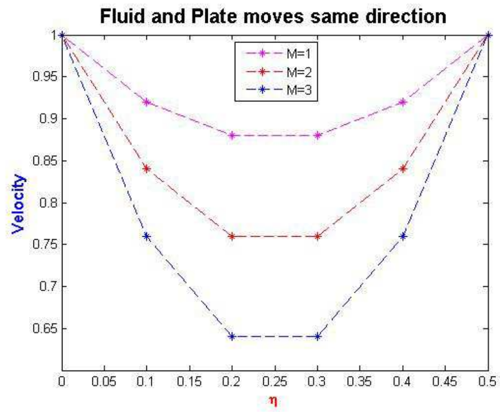

Figure 4: $f'(\eta)$ versus η for different values of M

Here we have shown the comparison of velocity and displacement when plate is stationary and plate and fluid moves in same direction with same velocity.

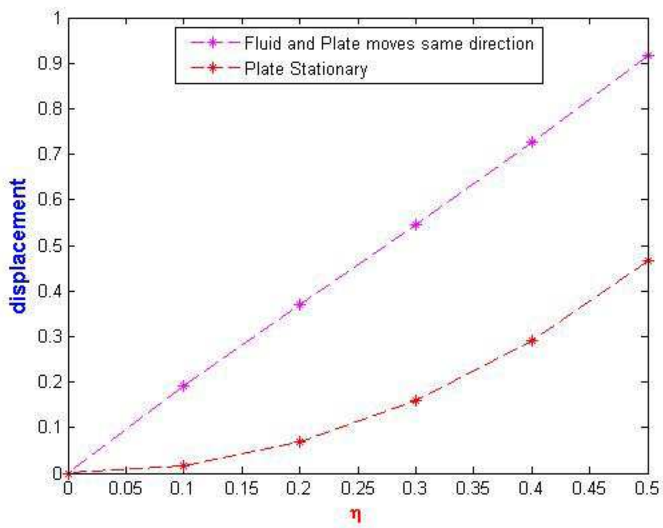

Figure 5: Comparison of $f(\eta)$ Versus η, when M=1

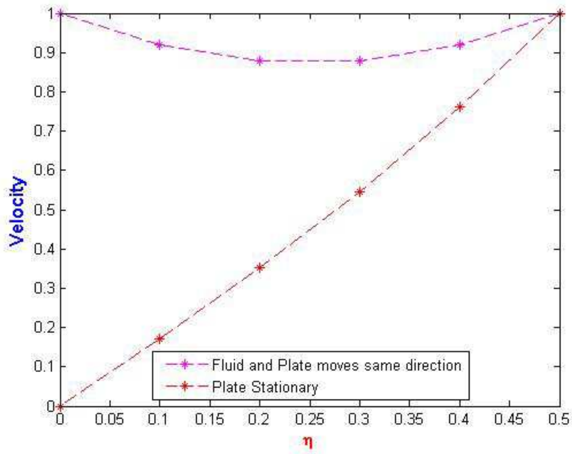

Figure 6: Comparison of $f'(\eta)$ Versus η, when M=1

4. RESULTS AND DISCUSSIONS

The result shows that, there is a significant impact of magnetic parameter on the displacement profile of the flow (Figure 1, 3). In both the cases we can see that, displacement of fluid increase exponentially but as magnetic parameter increases, displacement of fluid decreases. Figure 2 demonstrate that velocity of MHD power law Newtonian fluid is transversely proportional to magnetic parameter. When fluid and plate moves in same direction with same velocity, there is no change in velocity but increment in magnetic parameter shows remarkable change. Velocity profile undergoes decay as increase in magnetic parameter Figure 4. From figure 5, when plate is stationary, displacement of fluid is lesser than displacement of fluid when plate and fluid moves same direction. Comparisons of velocity profile in both the cases are given in figure 6.

5. CONCLUSIONS

In power-law MHD flow, there is remarkable effect of magnetic parameter on flow, whether the plate is at rest or fluid and plate moves. Magnetic parameter is transversely proportional to the velocity of fluid.

We find the generalization of blue method for third order problem and solved the problem using spline collocation method due to Blue. The beauty of this method is, there is no need to convert nonlinear problem into linear one, and we can get the solution directly for the nonlinear form. Thus, we can get the results for practical approach, without doing the experimental work and save the time and experimental cost.

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